

transverse and longitudinal velocity components; $p = p^*/\rho v_c$, dimensionless pressure; ρ , gas density; Pe , Peclet number; λ , thermal conductivity; c_p , heat capacity at constant pressure; $m = Pe r_c / T_c c_p$, dimensionless complex; $r_c = q/\rho v_c$, heat of sublimation; T_c , sublimation surface temperature; $T = T^*/T_c$, dimensionless temperature; T_0 , lower wall surface temperature; $Nu = H/\rho c_p$, Nusselt number.

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ROLE OF DROP SUPERHEATING IN THE NONSTEADY DROP COOLING OF A HIGH-TEMPERATURE WALL

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The influence of superheating and explosive boiling of a contact layer of drops on the heat transfer in drop cooling of a wall at a temperature exceeding the temperature of achievable heating of the liquid is considered.

Investigations of the interaction between a drop and a hot surface have mainly related to a large drop present on a horizontal surface for a relatively long time (a spheroidal state of the drop) [1]. In these conditions, the interaction process between the drop and the surface is steady, and the basic mechanism of heat transfer from the surface to the drop is heat transfer through a steady vapor layer. However, with short-term impact of the drop on the wall surface, no steady vapor layer is able to develop, and therefore its role may be insignificant. In the latter case, drop contact with the surface that is close to "liquid contact" may be realized. Then the heat conduction in the drop itself plays the basic role in the heat-transfer process. With sufficiently brief contact, impulse heating of the contact layer of the drop occurs; this leads to superheating and explosive effervescence of this layer. After effervescence, the wall layer becomes a finely disperse two-phase system, and then a vapor. At this stage, the intensive induced convection in the layer excited by explosive boiling plays an important role in heat transfer.

This nonsteady heat-transfer mechanism occurs in the initial stage of any contact between liquid and high-temperature solids. However, if the overall contact time is much larger than the time of heating and explosive boiling of the contact layer, the role of this mechanism is slight in comparison with the steady mechanism of heat transfer through the steady vapor layer. In the case of short-term contact, however, the nonsteady mechanism may play the fundamental role in heat transfer from a heated wall. If such conditions arise in the course of drop cooling, considerable increase in heat transfer in comparison with the heat transfer in steady conditions through the vapor layer may be expected on account of the above-noted intense heat-transfer mechanisms.

In describing drop cooling, no account is usually taken of drop superheating and the explosive boiling of the contact layer [1]. The aim of the present work is to rectify that omission to some degree.

1. Temperature and Time of Achievable Superheating of Liquid

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TABLE 1. Leidenfrost Temperature T_L and Temperature of Achievable Superheating T_{a0} of Liquids

Material	T_c	T_L	$T_{a0}[6]$	$\frac{T_L}{T_c}$	$\frac{T_{a0}}{T_c}$
	K				
Benzene	562,1	468 [5]	498	0,833	0,870
n-Heptane	540,2	485 [4]	486	0,898	0,900
n-Hexane	507,8	455 [4]	455	0,896	0,896
n-Pentane	469,7	419 [4]	419	0,892	0,891
Hexadecane	718,9	653 [5]	—	0,908	—
Isooctane	543,9	457 [5]	488	0,840	0,897
α -Methylnaphthalene	772	693 [5]	—	0,898	—
Ethyl alcohol	516,2	458 [5]	463	0,887	0,897
Water	647,3	575 [5]	575	0,888	0,888

The limit of existence of a metastable liquid (the spinodal) is determined from the equations of state of the material according to the condition $(\partial p/\partial V)_T = 0$. The Van der Waals equation gives underestimates of the temperature T_S of limiting superheating of the liquid in comparison with the experimental value. At zero pressure, $T_{S0} = 0.844T_c$, whereas the achievable superheating of organic liquids in experiments [2] $T_{a0} = 0.90T_c$. A more realistic position of the spinodal is given by the Bertholot equation: $T_{S0} = 0.919T_c$ [3]. In the vicinity of the spinodal, explosive boiling of metastable liquids with the appearance of vapor nuclei at the fluctuations is possible; this process determines the temperature of achievable superheating of the liquid T_a .

In experiments with drops on a hot horizontal surface, the Leidenfrost temperature T_L , corresponding to the maximum drop evaporation time, is determined from the temperature dependence of the drop evaporation time. At a surface temperature $T_w \geq T_L$, a steady vapor layer is established between the drop and the surface. In [4], it was shown that, for clean, smooth surfaces, T_L for a liquid is close to the temperature T_L of achievable liquid superheating at atmospheric pressure. Table 1 gives values of T_L from the data of [4, 5] and the experimental temperature of achievable superheating T_{a0} from [2, 6]. The mean value of the reduced Leidenfrost temperature $T_L/T_c = 0.882$; for the achievable superheating, $T_{a0}/T_c = 891$. For water, $T_L = T_{a0} = 575^\circ\text{K}$. The closeness of T_L and T_{a0} indicates that, in contact of a drop and a high-temperature surface, when $T_w \geq T_L$, explosive boiling of the contact layer of the drop is possible. For rough and contaminated surfaces, and in conditions of poor wettability, T_L is markedly lower than T_{a0} ; in this case, at contact between the drop of and the high-temperature surface, effervescence of the contact layer occurs by the heterogeneous-boiling mechanism. However, with increase in external pressure, T_L still approaches T_{a0} [7].

Superheating of the liquid to the temperature T_a at which explosive boiling is possible entails a sufficiently large heating rate corresponding to the heating time t_a . This time must be so small that the proportion of the liquid converted to vapor on account of evaporation from the heating surface in the heating process and on account of the formation of growth of vapor nuclei at the appropriate centers is small in comparison with the total volume of heated liquid. Table 2 gives values of the maximum heating time $t_{a, \max, 0}$ at normal atmospheric pressure for n-heptane, ethyl alcohol, and water in the liquid state; these values are obtained on the basis of generalizing the experimental and theoretical results of [2, 6].

2. Collision of Drop with Cold Wall.

Suppose that a drop of mass m and radius R moves along the normal to the wall at velocity v_0 , and that R and v_0 are such that the drop retains the form of a spherical segment with a spherical cross section in the contact area over the whole course of deformation. The possible flattening of a drop into a disk and breakdown into smaller elements is not considered. Change in radius of the sphere on impact is not taken into account, i.e., it is assumed that the Laplacian pressure inside the drop is always equal to $2\sigma/R = \text{const}$. Then the reaction force of the wall on drop impact is

$$F = -m \frac{dv}{dt} = -\frac{2\sigma}{R} S. \quad (1)$$

The instantaneous contact area S is equal to the area of a flattened segmentary surface, that is

TABLE 2. Parameters of Drop Cooling of High-Temperature Wall

Material		n-Heptane	Ethyl alcohol	Water
$t_{a, \max, 0}, \mu\text{sec}$ $R_{\max}, \mu\text{m}$		1000 303	1000 298	100 87
$We = 6,$ $R = R_{\max}$	$v_0, \text{m/sec}$	0,55	0,61	1,59
	$M, \%$	1,12	1,44	3,08
	$q_Q/q_m, \text{kJ/kg}$	8,8	30	95
$We = 80,$ $R = R_{\max}$	$v_0, \text{m/sec}$	2,01	1,97	5,79
	$M, \%$	4,11	4,66	11,2
	$q_Q/q_m, \text{kJ/kg}$	32	96	345
$We = 80,$ $R = 1 \mu\text{m}$	$v_0, \text{m/sec}$	35	34	54
	$M, \%$	17,1	19,4	34,2
	$q_Q/q_m, \text{kJ/kg}$	134	400	1050

$$S = 2\pi R x, \tag{2}$$

and hence

$$F = -4\pi\sigma x = -kx, \tag{3}$$

where $k = 4\pi\sigma$ is the rigidity coefficient of the drop.

It follows from Eqs. (1) and (3) that

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x = -\omega^2 x, \tag{4}$$

with the solution

$$x = x_{\max} \sin \omega t, \tag{5}$$

i.e., the drop is in contact with the surface according to a sinusoidal law, for which the frequency is

$$\omega = \left(\frac{k}{m}\right)^{1/2} = 2 \left(\frac{\pi\sigma}{m}\right)^{1/2}, \tag{6}$$

the period is

$$\tau = \left(\frac{\pi m}{\sigma}\right)^{1/2} \tag{7}$$

and the amplitude is

$$x_{\max} = \frac{v_0}{2} \left(\frac{m}{\pi\sigma}\right)^{1/2}. \tag{8}$$

The maximum value of the contact area is

$$S_{\max} = 2\pi R x_{\max} = \pi R v_0 \left(\frac{m}{\pi\sigma}\right)^{1/2}. \tag{9}$$

The value of τ from Eq. (7) is 1.63 times higher than the second-order free-oscillation period obtained in [8] for drops arising in the breakdown of liquid jets (ellipsoidal oscillations).

Suppose that the time t_{\max} of drop interaction with the wall is half τ ; then expressing the drop mass in terms of its radius, Eq. (7) gives

$$t_{\max} = \frac{1}{2} \tau = \frac{\pi}{3} \left(\frac{\rho}{\sigma}\right)^{1/2} R^{3/2}. \tag{10}$$

Analogously, Eqs. (8) and (9) are written in the form

$$x_{\max} = v_0 \left(\frac{\rho}{3\sigma} \right)^{1/2} R^{3/2}, \quad (11)$$

$$S_{\max} = 2\pi v_0 \left(\frac{\rho}{3\sigma} \right)^{1/2} R^{5/2}. \quad (12)$$

3. Interaction of Drop with Hot Wall

Suppose that the surface temperature of the wall is above the Leidenfrost temperature, that Eqs. (10)-(12) are valid for the impact of a drop moving over the normal to the wall, and that heat transfer between the drops and the surface occurs only in the time of mutual contact. Evaporation of the drop as it moves is neglected.

First, limits are set on the drop radii and velocity. In the model adopted here, drop deformation x_{\max} on impact must not exceed its radius R , i.e., its kinetic energy $mv_0^2/2$ must not exceed the potential energy of deformation $kR^2/2$. According to Eq. (3), this requirement is expressed by the inequality.

$$\frac{2\rho v_0^2 R}{\sigma} = We \leq 6. \quad (13)$$

Note, for comparison, that in experiments on the interaction of drops of radius $R = 1.15$ mm and velocity $v_0 = 1.08$ - 1.25 m/sec with a surface heated to 400°C [9], the following criterion for the absence of drop fractionation was established

$$\frac{2\rho v_{aw}^2 R}{\sigma} \leq We_c = 80, \quad (14)$$

where v_{aw} is the velocity of drop motion away from the surface, and is assumed to equal v_0 .

One further constraint on the drop radius is introduced taking account of the time of drop interaction with the surface in Eq. (10). Since steady heat transfer in a spheroidal state of the drop is excluded from consideration, it is assumed that the drop contact time t_{\max} is no greater than the maximum time $t_{a, \max, 0}$ of heating of the liquid to the temperature of explosive boiling at normal atmospheric pressure (Table 2).

This gives the following constraint for the drop radii

$$R \leq R_{\max} = \left(\frac{3}{\pi} \right)^{2/3} \left(\frac{\sigma}{\rho} \right)^{1/3} t_{a, \max, 0}^{2/3}. \quad (15)$$

The character of the thermal interaction between liquids and heated surfaces was investigated in [2] by the method of pulsed heating of wires immersed in liquid. Recordings obtained by the fast-photography method show that, on heating a wire by a current pulse of length no greater than $50 \mu\text{sec}$, the formation and growth of individual vapor nuclei at preexisting centers first occurs, but their presence does not have significant influence on the character of the thermal interaction of the liquid and the surface. Heat transfer is sharply intensified only after the liquid temperature in the wall layer approaches the temperature of achievable liquid superheating T_a .

Taking these results into account, it is assumed that, in the impact contact of a drop with a wall, heat transfer occurs by the following mechanism. In the initial stage of contact, there is superheating of liquid in the contact layer to a temperature T_a , followed by explosive effervescence of this layer. As a result of the explosion, the layer is converted to a two-phase finely disperse state. Since the process is nonequilibrium, the small liquid drops which are formed undergo intensive motion and rapidly evaporate on contact with the hot surface. The vapor layer which appears is heated in a short time by the mechanism of induced convection to a mean temperature $0.5(T_b + T_w)$.

To estimate the heat transfer from the surface, the mass fraction of the drop $M = \Delta m/m$ turned to vapor in the contact time must be calculated. If the thickness of the contact layer evaporated in δ and the area of the contact surface is given by Eq. (12), then

$$M = \frac{\Delta m}{m} = \frac{v_0 \delta}{2} \left(\frac{3\rho}{\sigma R} \right)^{1/2}. \quad (16)$$

The evaporated-layer thickness δ is estimated by two methods: by calculating the thickness of contact-layer heating to temperatures above the saturation temperature T_b (binodal point); and by calculating the radius of the vapor bubbles growing in this layer.

In the first method, contact is assumed to be purely liquid, and it is supposed that the heat is transferred by the mechanism of heat conduction in a liquid. The temperature distribution in the contact layer before explosive effervescence along the normal to the interface is expressed by the formula

$$T = T_a \exp \left[-\frac{x}{(at)^{1/2}} \right], \quad (17)$$

where T_a is the temperature of achievable superheating at the liquid boundary: $(at)^{1/2}$ is the characteristic heating depth of the liquid at time t .

The layer thickness δ of the metastable liquid, which is converted to a two-phase state after boiling, is found from Eq. (17) by the condition $x = \delta$ when $T = T_b$, that is

$$\delta = (at)^{1/2} \ln \frac{T_a}{T_b}. \quad (18)$$

Using the time $t = t_{\max}$ from Eq. (10), it is found that

$$\delta = \left(\frac{\pi}{3} \right)^{1/2} R^{3/4} a^{1/2} \left(\frac{\rho}{\sigma} \right)^{1/4} \ln \frac{T_a}{T_b}. \quad (19)$$

Then according to Eq. (16)

$$M = \frac{\Delta m}{m} = \frac{\pi^{1/2}}{2} v_0 R^{1/4} a^{1/2} \left(\frac{\rho}{\sigma} \right)^{3/4} \ln \frac{T_a}{T_b}. \quad (20)$$

The second method used the Plesset and Tsvikk formula for the time dependence of the vapor-bubble radius in the heated liquid [2]

$$r = 2 \left(\frac{3}{\pi} \right)^{1/2} \frac{(\lambda \rho c)^{1/2}}{L \rho'} (T - T_b) t^{1/2}. \quad (21)$$

Suppose that a vapor layer of thickness $2r$ is formed in the time of drop contact with the wall and that the mean temperature in the liquid T is $0.5(T_a + T_b)$. Then the proportion of liquid evaporating in the drop in the contact time, according to Eqs. (10), (12), (21), is

$$M' = \frac{\Delta m'}{m} = 3^{1/2} v_0 R^{1/4} a^{1/2} \left(\frac{\rho}{\sigma} \right)^{3/4} \frac{c(T_a - T_b)}{L}. \quad (22)$$

The only difference between Eqs. (20) and (22) is in the numerical factors and the expressions for the metastability factor. The ratio

$$\frac{M}{M'} = \frac{\Delta m}{\Delta m'} = \frac{(3\pi)^{1/2}}{2} \frac{L \ln(T_a/T_b)}{c(T_a - T_b)}$$

for water is 1.24, i.e., Eqs. (20) and (22) give practically the same values for the mass fraction of the drop evaporating in the course of contact with the wall.

For the specified Weber number We , Eq. (20) is written in the form

$$M = \frac{1}{2} \left(\frac{\pi We}{2} \right)^{1/2} a^{1/2} \left(\frac{\rho}{R\sigma} \right)^{1/4} \ln \frac{T_a}{T_b}, \quad (23)$$

from which it follows that M increases with decrease in drop radius.

The heat Q_1 transmitted to the drop by the wall in the course of nonequilibrium transition of the contact layer to vapor consists of three terms: the component heating the stable and metastable liquid; the heat of preevaporation of drops in the two-phase system; and the component heating the vapor to the temperature $0.5(T_b + T_w)$. This heat is close in value to the equilibrium heat of formation of the vapor layer; therefore, the following expression is written

$$Q_1 = \Delta m [c(T_b - T_0) + L + 0,5c'(T_w - T_b)]. \quad (24)$$

The heat-flux density removed from the wall by the drop flux in drop motion along the normal is

$$q_Q = q_m M [c(T_b - T_0) + L + 0,5c'(T_w - T_b)], \quad (25)$$

where q_m is the density of the mass flux in a drop flow.

The mean value of the heat-transfer coefficient per unit mass-flux density is

$$\frac{\alpha}{q_m} = \frac{q_Q}{q_m(T_w - T_0)}. \quad (26)$$

Table 2 gives values of the maximum time $t_{a, \max, 0}$ of liquid heating to the temperature of explosive boiling according to the estimates of [2, 6], the maximum radius R_{\max} of an intact drop according to Eq. (15), and the corresponding velocity v_0 according to Eqs. (13) and (14), for $We = 6$ and 80. Table 2 also gives values of the initial velocity of a drop of radius $R = 1 \mu\text{m}$ for $We = 80$. The proportion of the evaporated mass fraction of the drop for the given R and We in Table 2 are calculated from Eq. (20). For these radii and velocities, the ratio of the heat-flux density q_Q and the mass-flux density q_m in the drop flux is found from Eq. (25), for a wall temperature $T_w = 1000^\circ\text{K}$ and an initial drop temperature $T_0 = 298^\circ\text{K}$. Calculations for $We = 80$ are approximate in character.

4. Discussion of Results

It follows from Eqs. (20) and (22) that the fraction of evaporated mass of the drop in brief contact with the high-temperature wall is proportional to its velocity; therefore, the drop velocity is the determining factor in the nonsteady heat transfer in drop cooling of the wall. This confirms the analogous conclusion in [10], based on direct experimental measurements of the heat removal produced by individual water drops of radius 100–200 μm at velocities of 2.4–10 m/sec [10].

At constant velocity v_0 , M is proportional to $R^{1/4}$; according to Eq. (23), for water

$$M = 0,2v_0 R^{1/4} (R, \text{m}; v_0, \text{m/sec}). \quad (27)$$

At constant Weber number, conversely, M increases with decrease in drop radius according to Eq. (24). In this case, for water

$$M = 12 \cdot 10^{-4} We^{1/2} R^{-1/4}. \quad (28)$$

In particular, for water in critical conditions, when $We = We_c = 80$, $M = 11 \cdot 10^{-3} R^{1/4}$ (R, m). In these conditions, decreasing the radius from $R = R_{\max}$ to $R = 1 \mu\text{m}$ increases M for *n*-heptane, ethyl alcohol, and water by a factor of 3–4 (Table 2).

In [10], values of the parameter ϵ , the ratio of the heat removed from the surface by the drop to the heat involved in heating the drop to the boiling point and its total evaporation were given. The parameters ϵ and M are related as

$$\frac{\epsilon}{M} = \frac{Q_1}{\Delta m [c(T_b - T_0) + L]}.$$

These parameters coincide if the third term in brackets is neglected in Eq. (24) for Q_1 .

An increase in ϵ from 3 to 25% when the velocity increases from 2.4 to 10.2 m/sec was established in the experiments of [10] from recordings obtained by high-speed photography for

drops of radius $R = 135 \mu\text{m}$ at a wall temperature $T_w = 620^\circ\text{C}$; for drops of larger radius $R = 200 \mu\text{m}$, $\epsilon = 16\%$ at $v_0 = 10.2 \text{ m/sec}$. An analogous dependence of ϵ on v_0 was obtained in [11] with drop cooling of the surface at $T_w = 900\text{--}1200^\circ\text{K}$. According to [11], ϵ increases from 5 to 7% with increase in drop velocity from 11 to 32 m/sec for a mass-flux density of atomized water $q_m = 4 \text{ kg/m}^2\text{sec}$. Thus, the conclusion drawn here that heat transfer is intensified with increase in drop velocity in the case of nonsteady interaction with a hot surface is confirmed by the experimental results of [10, 11].

In the experiments of [9], $\epsilon = 0.15\%$ was found for water drops with $R = 1150 \mu\text{m}$, $v_0 = 1.08\text{--}1.47 \text{ m/sec}$, $T_w = 349^\circ\text{C}$. In these experiments, the contact time of the drop was much larger than the maximum time $t_{\alpha, \text{max}, 0}$ of liquid superheating to the temperature of explosive boiling (Table 2); therefore, heat transfer was basically by a steady mechanism in conditions of a spheroidal state of the drop and hence these data are not comparable with the values of M given in the present work.

According to Eqs. (20) and (25), the ratio $\alpha/q_m = 490 \text{ J/kg}\cdot\text{K}$ for critical conditions at $T_w = 1000^\circ\text{K}$ and $We = We_c = 80$, $R = R_{\text{max}}$ (Table 2); for $We = 6$, $R = R_{\text{max}}$, $\alpha/q_m = 135 \text{ J/kg}\cdot\text{K}$. In the experiments of [11] with drop cooling of the metal with a surface temperature $T_w = 900\text{--}1200^\circ\text{K}$ at a drop velocity of 11–32 m/sec in the range $q_m = 0.3\text{--}9.0 \text{ kg/m}^2\cdot\text{sec}$, the ratio $\alpha/q_m = 130\text{--}160 \text{ J/kg}\cdot\text{K}$, which is comparable with the results of our present experiments. Note that, in real conditions, as a result of collisions between drops and deviation in their direction from the normal, α/q_m must be lower than the result given by Eqs. (20) and (25).

The foregoing estimates lead to practical conclusions regarding how the drop cooling of a surface with a temperature exceeding the temperature of achievable superheating of the liquid may be intensified. This entails using drop fluxes with a mean drop radius given by Eq. (15); the mean Weber number We for the drops in the flux must be somewhat below the critical value We_c . Then, according to Eqs. (23)–(26), increasing q_0/q_m and α/q_m entails decreasing the drop radius and simultaneously increasing the drop velocity. With increase in spraying flux density, q_m , q_0 and α at first increase in proportion to q_m ; at large q_m , this increase is slowed on account of mutual collision of the drops.

NOTATION

p , pressure; V , volume, T , temperature; T , Leidenfrost temperature; t , time; R , drop radius; v_0 , initial drop velocity; v , instantaneous velocity of the drop center of mass; m , drop mass; σ , surface tension; F , force; S , area; x , drop deformation; k , drop rigidity coefficient; ω , angular frequency; τ , oscillation period; ρ , density; We , Weber number; M , ratio of evaporated drop mass to initial drop mass; δ , thickness of evaporated contact layer of drop; a , thermal diffusivity; λ , thermal conductivity; c , specific heat; L , specific heat of vaporization; r , radius of vapor bubble; Q_1 , heat transmitted by a single drop; q_0 , heat flux density; q_m , mass flux density in droplet spraying; α , heat-transfer coefficient; ϵ , ratio of the heat transmitted to the drop to the sum of the heat involved in heating the drop to the boiling point and in complete evaporation, Indices: s , spinodal; c , critical; a , achievable superheating; 0 , initial, atmospheric pressure; w , wall, surface; max , maximum.

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NUMERICAL ANALYSIS OF TRANSFER PHENOMENA IN SEMICONDUCTOR DEVICES AND STRUCTURES.

1. UNIVERSAL PROGRAM FOR TWO-DIMENSIONAL MODELING

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A universal program for the two-dimensional numerical analysis of functionally integrated structures of integral circuits is described.

An important instrument of profound theoretical research of semiconductor devices and elements of integral circuits is comprised by programs of multidimensional numerical analysis. Their distinguishing feature is the possibility of analyzing structures of various design with an arbitrary number of p-n junctions. At present, several reports on programs of two-dimensional numerical analysis of this type have already appeared [1, 2]. However, according to existing information, it is fairly difficult to judge their potential and efficiency. The development of universal programs of three-dimensional analysis is evidently inexpedient at this point, because of the enormous time required to model even the simplest structures using current supercomputers [3]. Overall, the complexity of developing such programs depends primarily on a series of interrelated problems of methodological type: 1) the lack of universal procedures for choosing the initial approximation; 2) the low efficiency and reliability of well-known methods of numerical solution of the fundamental system of semiconductor-physics equations, including discretization methods.

In the present work, a universal program of two-dimensional numerical analysis of elements of PNAIIL integral circuits is described. To a definite extent, the above-noted complexities may be solved in developing this program. The results of computational experiments on EC-series computers are given, illustrating the possibilities of the program and its efficiency.

Description of the Program

Using the PNAIIL program, the fundamental system of semiconductor-physics equations is solved numerically in the two-dimensional approximation [4, 5]

$$\varepsilon \nabla^2 \psi = -q(p - n + N_d - N_a), \quad (1)$$

$$\nabla \vec{j}_p = -qR, \quad \vec{j}_p = -q\mu_p p \nabla \Phi_p, \quad (2)$$

$$\nabla \vec{j}_n = qR, \quad \vec{j}_n = -q\mu_n n \nabla \Phi_n \quad (3)$$

with the auxiliary relations

$$\begin{aligned} n &= n_i \gamma_n \exp(A \Delta V_g / \varphi_T) \exp[(\psi - \Phi_n) / \varphi_T], \\ p &= n_i \gamma_p \exp[(1 - A) \Delta V_g / \varphi_T] \exp[(\Phi_p - \psi) / \varphi_T]. \end{aligned} \quad (4)$$

The boundary condition models invoked are standard [3]. Provision is made for the use of the following models as the initial model information: 1) the Caughey-Thomas model [6] for mobilities μ_n and μ_p ; 2) the Shockley-Reid-Hall model [7] and Auger recombination [8] for the

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